1987 - AB1

1. A particle moves along the x-axis so that its acceleration at any time t is given by $a(t) = 6t - 18$. At time $t = 0$ the velocity of the particle is $v(0) = 24$, and at time $t = 1$ its position is $x(1) = 20$

(a) Write an expression for the velocity $v(t)$ of the particle at any time t.

(b) For what values of t is the particle at rest?

(c) Write an expression for the position $x(t)$ of the particle at any time t.

(d) Find the total distance traveled by the particle from $t = 1$ to $t = 3$.

1987 - AB2

2. Let $f(x) = \sqrt{1 - \sin x}$.

(a) What is the domain of f?

(b) Find $f'(x)$.

(c) What is the domain of f'?

(d) Write an equation for the line tangent to the graph of f at $x = 0$.

1987 - AB3

3. Let R be the region enclosed by the graphs of $y = (64x)^{\frac{1}{4}}$ and $y = x$.

(a) Find the volume of the solid generated when region R is revolved about the <u>x-axis</u>.

(b) Set up, but <u>do not integrate</u>, an integral expression in terms of a single variable for the volume the solid generated when region R is revolved about the <u>y-axis</u>.

1987 – AB 4

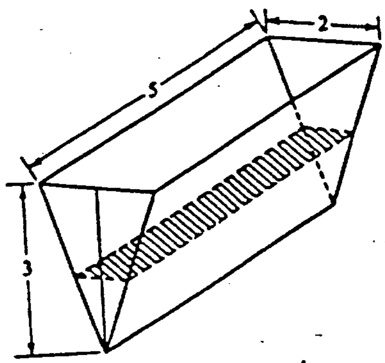
4. Let f be the function given by $f(x) = 2\ln(x^2+3) - x$ with domain $-3 \leq x \leq 5$.

(a) Find the x-coordinate of each relative maximum point and each relative minimum point of f. Justify your answer.

(b) Find the x-coordinate of each inflection point of f.

EXAMINE W/ CALCULATOR

(c) Find the absolute maximum value of $f(x)$.



1987 – AB 5

5. The trough shown in the figure above is 5 feet long, and its vertical cross sections are inverted isoscele triangles with base 2 feet and height 3 feet. Water is being siphoned out of the trough at the rate of 2 cubic feet per minute. At any time t, let h be the depth and V be the volume of water in the trough.

(a) Find the volume of water in the trough when it is full.

(b) What is the rate of change in h at the instant when the trough is $\frac{1}{4}$ full by volume?

(c) What is the rate of change in the area of the surface of the water (shaded in the figure) at the instant when the trough is $\frac{1}{4}$ full by volume?

1987 – AB 6

6. Let f be a function such that $f(x) < 1$ and $f'(x) < 0$ for all x.

(a) Suppose that $f(b) = 0$ and $a < b < c$. Write an expression involving integrals for the area of the region enclosed by the graph of f, the lines $x = a$ and $x = c$, and the x-axis.

(b) Determine whether $g(x) = \dfrac{1}{f(x) - 1}$ is increasing or decreasing. Justify your answer.

(c) Let h be a differentiable function such that $h'(x) < 0$ for all x. Determine whether $F(x) = h(f(x))$ is increasing or decreasing. Justify your answer.

1987 - BC 1

At any time $t \geqq 0$, in days, the rate of growth of a bacteria population is given by $y' = ky$, where k is a constant and y is the number of bacteria present. The initial population is 1,000 and the population triples during the first 5 days.

(a) Write an expression for y at any time $t \geqq 0$.

(b) By what factor will the population have increased in the first 10 days?

(c) At what time t, in days, will the population have increased by a factor of 6?

1987 - BC 2

2. Consider the curve given by the equation $y^3 + 3x^2y + 13 = 0$.

(a) Find $\frac{dy}{dx}$.

(b) Write an equation for the line tangent to the curve at the point $(2, -1)$.

(c) Find the minimum y-coordinate of any point on the curve. Justify your answer.

1987 - BC 3

3. Let R be the region enclosed by the graph of $y = \ln x$, the line $x = 3$, and the x-axis.

(a) Find the area of region R.

(b) Find the volume of the solid generated by revolving region R about the x-axis.

(c) Set up, but do not integrate, an integral expression in terms of a single variable for the volun the solid generated by revolving region R about the line $x = 3$.

1987 - BC 4

4. (a) Find the first five terms in the Taylor series about $x = 0$ for $f(x) = \frac{1}{1 - 2x}$.

(b) Find the interval of convergence for the series in part (a).

(c) Use partial fractions and the result from part (a) to find the first five terms in the Taylor series about $x = 0$ for $g(x) = \frac{1}{(1 - 2x)(1 - x)}$.

1987 - BC 5

5. The position of a particle moving in the xy-plane at any time t, $0 \leqq t \leqq 2\pi$, is given by the parametric equations $x = \sin t$ and $y = \cos(2t)$.

(a) Find the velocity vector for the particle at any time t, $0 \leqq t \leqq 2\pi$.

(b) For what values of t is the particle at rest?

(c) Write an equation for the path of the particle in terms of x and y that does not involve trigonometric functions.

(d) Sketch the path of the particle in the xy-plane below.

1987 - BC 6

6. Let f be a continuous function with domain $x > 0$ and let F be the function given by $F(x) = \int_1^x f(t)dt$ for $x > 0$. Suppose that $F(ab) = F(a) + F(b)$ for all $a > 0$ and $b >$ and that $F'(1) = 3$.

(a) Find $f(1)$.

(b) Prove that $aF'(ax) = F'(x)$ for every positive constant a.

(c) Use the results from parts (a) and (b) to find $f(x)$. Justify your answer.